Effect of Fluid Drag on Low Reynolds Number Discharge of Solids from a Circular Orifice

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A dimensionless solids flowrate for discharge from an orifice in the bottom of a containing vessel is shown to be a function of a parameter that measures the importance of fluid drag to particle inertia. The analysis assumes a linear relation between pressure gradient and slip velocity and compares favorably to experiment within its supposed region of applicability for three experimental situations: the self-induced, air-impeded discharge of fine particles from a conical hopper; the discharge of particles from a sealed egg-timer-like vessel; and the air-assisted discharge of particles through an orifice.

SCOPE

The gravity discharge of granular material through an aperture in the base of a containing vessel is a common occurrence in mineral and food processing, coal handling, chemical production and farming. The rate of discharge is independent of the amount of material in the vessel, for material columns that are tall compared to the characteristic dimension across the vessel, because most of the weight of the material is supported by the vessel walls. Dimensional analysis then suggests that the solids flowrate depends only on the bulk density, the gravitational acceleration, and a characteristic length associated with the aperture and particle sizes (Crewdson et al., 1977; Brown and Richards, 1970). Thus, $\dot{m}_p/\rho_s(1-\epsilon)g^{1/2}L^{5/2})$ is a constant for a particular angle of approach to a circular aperture.

Experiments show that few particles flow through an annulus, bounded by the orifice edge, of width of the order of the particle diameter, d_p. This "statistically empty" annulus may be neglected for small d_p/D, so then we should have $L = D$.

When the above dimensionless flowrate is a function of the angle of approach only, there is evidence that the fluid occupying the void volume exerts no influence on the motion of the particles (Crewdson et al., 1977). If there is a drag force, which acts on the particles due to a slip velocity between the particle and fluid phases, and which is a significant percentage of the particle inertia force, the dimensionless flowrate should be a function of the ratio of these two forces in addition to the angle of approach.

In this paper, we investigate this notion in an effort to develop it to its logical end. Little information is available on the details of the interaction between a fluid and the solid phase during the gravity flow of solids, although fluid flow may be used to improve solids flow and, left unattended, may impede it (Resnick et al., 1966; Kurz and Rumpf, 1975; Crewdson et al., 1977). Our analysis is an extension of the Minimum Energy Theorem of Brown (1961), a theory that derives from an application of the First and Second Laws of Thermodynamics to the flow of granular material through an aperture. Implications of the new theory are examined with reference to the experimental results of Crewdson et al. (1977), Resnick et al. (1966) and Dhoka (1970).

CONCLUSIONS AND SIGNIFICANCE

Brown's Minimum Energy Theorem (Brown, 1961) for the discharge of solids from orifices can be extended to include the effect of the drag of an interstitial fluid. The dimensionless flowrate σ, where

$$\sigma = \frac{4\dot{m}_p \sin^{5/2}\beta}{\rho_s\pi(1-\epsilon)g^{1/2}(D-k)^{5/2}\frac{2}{3}(1-\cos^{3/2}\beta)}$$

depends monotonically on a parameter, γ, which represents the ratio of fluid drag to particle inertia. For a linear drag law, γ is proportional to the slip velocity, and is zero when the particles merely carry along the interstitial fluid. Negative values of γ imply a drag toward the orifice, which increases σ.

Comparison of the theory with experimental results is encouraging for three types of flow.

- Self-Induced, Air-Impeded Discharge of Fine Particles from Conical Hoppers. In this case, satisfactory agreement requires that both the void fraction, ϵ, and the volume flowrate of interstitial air, α, decrease with increasing particle size, but there is an unresolved effect of orifice diameter.
- Effect of Gravity on the Discharge of Particles from a Sealed Egg-Timer-Like Vessel. Here, the theory correctly predicts the results for large (360 μm) and small (90 μm) particles if ϵ is taken to be 0.5.
- Air-Assisted Discharge of Particles through Orifices in a Flat-Bottom Container. In this case, the results are satisfactorily correlated as σ against γ regardless of the choice of ϵ. Agreement with theory is satisfactory for $\epsilon = 0.5$ even for particle Reynolds numbers of order 100.

In view of the above agreement, the theory may be used in its range of applicability to estimate the solids discharge rate from the bottom of a containing vessel when fluid drag on the discharging particles influences the discharge rate. Further experimental studies that need to be carried out include *in situ* measurement of the void fraction for both positive and negative values of γ and the effect of large particle Reynolds numbers where fluid inertia effects are important.

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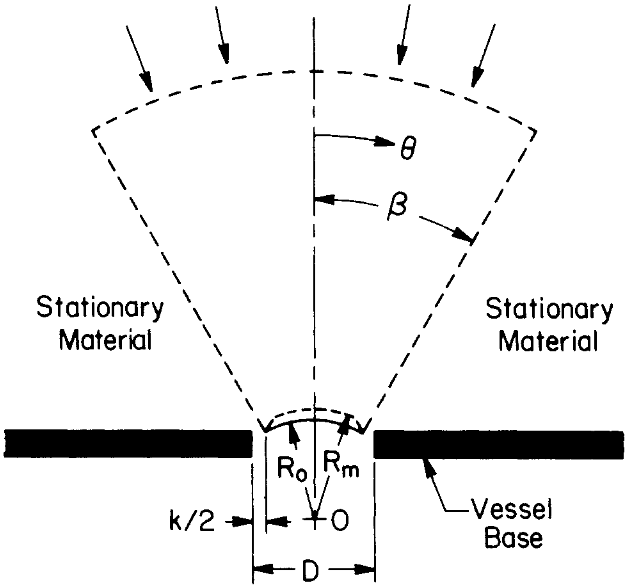

Figure 1. Conical section through which granular material falls toward a circular orifice of diameter D in the base of a containing vessel.

THEORETICAL DEVELOPMENT

In his Minimum Energy Theorem, Brown (1961) postulates that the sum of the kinetic and potential energies of the particles decreases along a streamtube in the direction of particle flow until it is minimized at the free-fall arch. From this postulation, an expression for the particle mass flowrate can be developed. We extend below Brown's postulation to include interaction between the particle and fluid phases.

The flow of interest takes place in a cone above a circular orifice as shown in Figure 1. If the vessel walls are located at an angle θ that is greater than β for the material of interest, β defines the location of a surface of sliding that separates moving from stationary granular material. If the wall angle θ is less than β, the vessel walls define the cone in which the flow takes place, and no stationary material is present in the vessel.

We will assume throughout this paper that the particle flow is radial toward the orifice and that the bulk density of the granular material can be taken as a constant. The first assumption is consistent with experimental results (Brown and Richards, 1970) while the second requires that the void fraction, ϵ, remain constant. It is well known that ϵ varies in the types of flows we are considering here. However, variations in ϵ are usually small compared to ϵ itself, and to include these variations would require a more complete theory than the one presented here. Then the ϵ variation would be computed as part of the solution to the problem.

Under the assumption of radial flow, an arbitrary streamtube lies along a ray, at an angle θ, emanating from the virtual origin o. The First Law of Thermodynamics for any such streamtube may be written as:

$$-d\left(\frac{P}{\rho} + \frac{v^2}{2} + gr\cos\theta\right) = du - \delta q - \delta w \qquad (1)$$

where differentials are taken along the streamtube, and the solids flow is toward the virtual origin. The work, w, in Eq. 1 is that done on the streamtube per unit mass of mixture by forces other than those due to normal stresses. δ implies an inexact differential.

We take both the granular material and the fluid to be incompressible, so $du = Tds$. We presume that the flow is irreversible, implying that $du - \delta q$ is larger than zero, and, in agreement with Brown (1961), that the right hand side of Eq. 1 is larger than zero so that the mechanical energy decreases along a streamtube in the direction of flow.

Below the free-fall arch, the particles fall freely, collisions are infrequent, and the flow, for all practical purposes, can be considered reversible so that a Bernoulli equation applies. Thus, the mechanical energy is a minimum at the free-fall arch located at radius R_m, Figure 1.

Following Murray (1965) and Soo (1967) we define the kinetic energy and pressure in Eq. 1 such that:

$$\rho v^2 = \rho_s(1 - \epsilon)v_s^2 + \rho_f \epsilon v_f^2 \qquad (2)$$

$$P = (1 - \epsilon)P_s + \epsilon P_f \qquad (3)$$

Since the mixture density, ρ, is:

$$\rho = \rho_s(1 - \epsilon) + \rho_f \epsilon \qquad (4)$$

and $\rho_f/\rho_s << 1$, the kinetic energy term in Eq. 1 can be associated with the particles alone, and ρ can be replaced by $\rho_s(1 - \epsilon)$.

If the fluid flow—either that induced by the particle motion or provided independent of it—is such that the particles are somewhat lubricated and nearly fluidized, then the particles are not capable of transmitting a substantial pressure, and P should be defined in the limit as $\epsilon \rightarrow 1$ so that $P = P_f$. Even if the fluid phase exerts little influence on the solid phase we are justified in removing P_s from Eq. 3 because, as Brown (1961) points out, the solid phase does not transmit pressure far due to the formation of "arches and to frictional forces between granules."

In view of the above discussion, the mechanical energy, e, where:

$$e = \frac{P}{\rho_s(1 - \epsilon)} + \frac{v^2}{2} + gr\cos\theta \qquad (5)$$

should be a minimum at R_m. In Eq. 5, we have dropped the subscripts on P and v. In what follows, P is the fluid-phase pressure, and v is the solid-phase speed.

The assumption of radial flow results in:

$$v = \frac{\lambda(\theta)}{r^2} \qquad (6)$$

where $\lambda(\theta)$ is a function to be determined. Eqs. 5 and 6, along with the result that e should be a minimum at $r = R_m$ yield:

$$0 = \frac{1}{\rho_s(1 - \epsilon)} \left.\frac{\partial P}{\partial r}\right|_{r=R_m} - \frac{2\lambda^2(\theta)}{R_m^5} + g\cos\theta \qquad (7)$$

For small particle Reynolds numbers the pressure gradient is proportional to the slip velocity. An approximate expression for the slip velocity can be developed by considering expressions for the particle and gas volume flowrates, $\dot{m}_p/\rho_s$ and $\dot{m}_f/\rho_f$, respectively:

$$\dot{m}_p/\rho_s = \int_o^\beta (1 - \epsilon)\, 2\pi\lambda(\theta) \sin\theta \, d\theta \qquad (8)$$

$$\dot{m}_f/\rho_f = \int_o^\phi \epsilon 2\pi\eta(\theta) \sin\theta \, d\theta \qquad (9)$$

where $\eta(\theta)$ plays the same role for the gas as $\lambda(\theta)$ does for the particles.

In Eq. 9, we have assumed that the gas flow is radial, so the gas speed is $\eta(\theta)/r^2$, and it is contained within a cone of half angle ϕ. If we take ϕ equal to β and define α to be the ratio of the gas to the particle volume flowrates, then one approximation that can be made is:

$$\eta(\theta) = \alpha \frac{1 - \epsilon}{\epsilon} \lambda(\theta) \qquad (10)$$

so the pressure gradient is:

$$\frac{\partial P}{\partial r} = -F(\epsilon)\left\{\frac{\alpha(\epsilon - 1) + \epsilon}{\epsilon} \frac{\lambda(\theta)}{r^2}\right\} \qquad (11)$$

where the term in brackets represents the slip velocity. For the proportionality factor $F(\epsilon)$ we choose (Kunii and Levenspiel, 1969):

$$F(\epsilon) = \frac{K\mu}{d_p^2}\left(\frac{1-\epsilon}{\epsilon}\right)^2 \tag{12}$$

where the dimensionless constant, K, is to be determined from experiment.

An expression for the mass flowrate of particles can be obtained by using Eq. 11 in Eq. 7, solving for $\lambda(\theta)$ and then applying Eq. 8. The result is:

$$\sigma = \frac{(\gamma^2+1)^{3/2} - (\gamma^2+\cos\beta)^{3/2} + \frac{3}{2}\gamma(\cos\beta - 1)}{(1-\cos^{3/2}\beta)} \tag{13}$$

where

$$\sigma = \frac{4\dot{m}_p \sin^{5/2}\beta}{\rho_s\pi(1-\epsilon)g^{1/2}(D-k)^{5/2}\frac{2}{3}(1-\cos^{3/2}\beta)} \tag{14}$$

and

$$\gamma = \frac{F(\epsilon)R_o^{1/2}}{\sqrt{8}\,\rho_s(1-\epsilon)g^{1/2}}\left\{\frac{\alpha(\epsilon-1)+\epsilon}{\epsilon}\right\} \tag{15}$$

To obtain Eq. 13, we set $R_m = R_o$ as an approximation (Brown, 1961).

The parameter σ is the dimensionless flowrate mentioned in the Conclusions and Significance section, and γ is proportional to the force ratio mentioned there. The significance of γ is made more clear when it is rewritten as

$$\gamma = \left\{\frac{K\mu}{\sqrt{8}\,\rho_s(1-\epsilon)(gd_p)^{1/2}d_p}\right\}\left(\frac{R_o}{d_p}\right)^{1/2}\left(\frac{1-\epsilon}{\epsilon}\right)^2\left\{\frac{\alpha(\epsilon-1)+\epsilon}{\epsilon}\right\} \tag{16}$$

so that γ is seen to be proportional to an inverse Reynolds number (the term in the first set of brackets). For small γ, the drag on the particles is small compared to the particle inertia, and the fluid phase should not influence the particle flowrate substantially; that is, for $\gamma = 0$, $\sigma = 1$. This is precisely the problem considered by Brown (1961).

Figure 2, a plot of σ vs. γ for two rather extreme values of β, shows that β does not have a strong influence on the dimensionless flowrate. Values of $\gamma < 0$ imply that the speed of the interstitial fluid toward the orifice is greater than that of the particles and hence drags them along. This situation leads to values of σ greater than unity and occurs for $\alpha > \epsilon/(1-\epsilon)$, i.e., when the volume of fluid moving with the particles is greater than the void volume. For $0 < \alpha < \epsilon/(1-\epsilon)$ the fluid flow is still toward the orifice, but the speed is less than that of the particles, and a drag force opposes the particle motion resulting in $\sigma < 1$. Fluid flow in a direction opposed to the particle motion occurs for $\alpha < 0$ and yields still smaller values of σ.

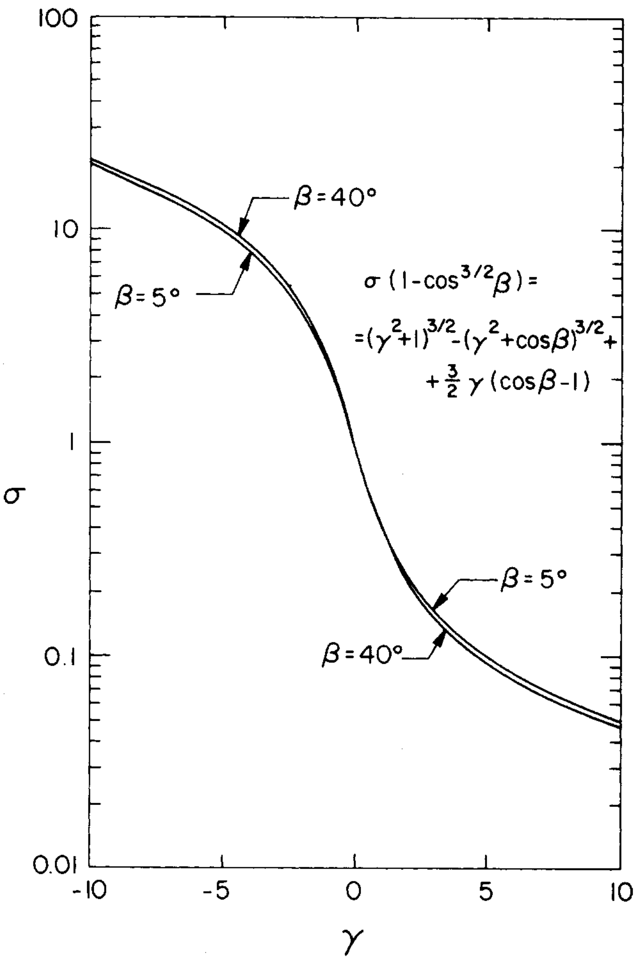

Figure 2. Theoretical prediction of σ versus γ.

APPLICATIONS OF THE THEORY

We will apply below the results of the previous section to three existing sets of experimental data. We hope to establish that the parameters developed above are the correct ones with which to correlate mass flowrate data from hoppers when the influence of the fluid phase cannot be neglected. The three flows are somewhat different from each other. In the first situation we consider, Crewdson et al. (1977) presumed that a gas flow, self-induced by particles discharging from a hopper, was in the direction of the particle flow but acted to retard the particle motion. Here we expect $\alpha < \epsilon/(1-\epsilon)$ and $\sigma < 1.0$. In the second flow, Dhoka, in an unpublished thesis (Dhoka, 1970) reported flowrate measurements for glass beads falling in an egg-timer-like vessel. Because the container is completely closed, downward particle motion induces an upward gas flow that again retards the particle motion and should result in σ values less than unity. Finally, we consider the results of Resnick et al. (1966) who studied the flow of sand from a hopper in which an air flow in the direction of the particle flow was artificially introduced. Generally we expect σ values from Resnick's experiment to exceed unity.

Crewdson et al. (1977) studied the discharge of fine particles of silica sand from conical hoppers with a 5.9° half angle. Because this angle is substantially less than the angle of approach, we can take β to be equal to 5.9°. The particles were sieved to diameters ranging from 98 μm to 780 μm, and the hoppers were open to the atmosphere on both entry and discharge ends. Measurements reported include mass flowrates, voidage distributions, and wall pressure distributions for four discharge orifice diameters ranging from 4 to 10 mm.

In the Crewdson case, a pressure distribution is set up in the interstitial gas as a result of the normal stress distribution (solids pressure) in the flowing particles. The solids pressure decreases slightly as the orifice is approached resulting in a small increase in ϵ over what it would be in the absence of flow acceleration. This increased void volume is not filled rapidly enough, due to flow resistance, to maintain atmospheric pressure, and the pressure above the orifice is below atmospheric (Kurz and Rumpf, 1975). This effect leads to a gas flowrate that is less than it would be if the interstitial gas were simply carried along with the discharge flow, at zero relative velocity. If that were the case, the interstitial gas pressure would be everywhere atmospheric, because dynamic pressure changes due to gas acceleration are extremely small. Hence, we expect to find $\alpha(1-\epsilon)/\epsilon$ in the range from zero to one.

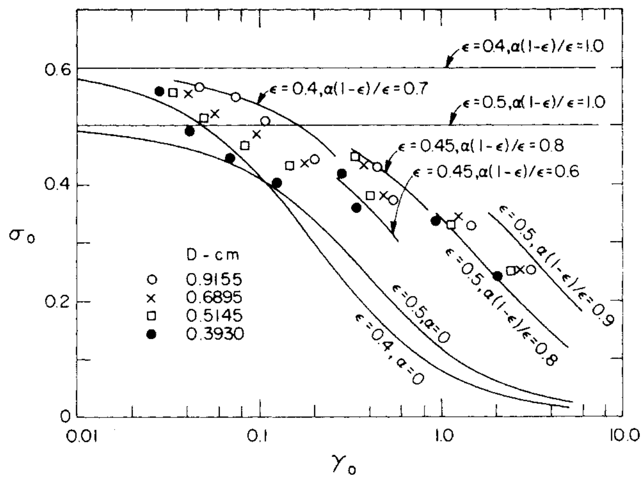

Figure 3. Influence of interstitial gas on the flow of silica sand from a conical hopper. Discrete data are those of Crewdson et al. (1977). Theoretical curves may be used to establish approximate values of ϵ and α for certain ranges of γ_o.

Crewdson et al. determined the void fraction distribution *in situ*, but after-the-fact, from measurements of the pressure distribution in a hopper in which the flow had been stopped gently. The results show considerable scatter near the orifice, but they indicate that ϵ increases and then decreases as the orifice is approached and decreases as the particle size increases. Crewdson's values of ϵ at the location of the free-fall arch were: 0.502 for d_p = 90-106 μm; 0.455 for d_p = 212-300 μm and 0.414 for d_p = 355-422 μm. These were the only data reported, and they must be regarded of uncertain accuracy.

Figure 3 shows the data from Figure 1 of the paper by Crewdson et al. (1977) for the eight particle sizes of 95, 139, 230, 252, 373, 500, 644, and 755 μm plotted as σ_o against γ_o where:

$$\sigma_o = (1 - \epsilon)\sigma \tag{17}$$

and

$$\gamma_o = \frac{\epsilon^2}{(1 - \epsilon)} \left\{\frac{\alpha(\epsilon - 1) + \epsilon}{\epsilon}\right\}^{-1} \gamma \tag{18}$$

In these expressions, σ_o and γ_o are defined so as to remove from σ and γ dependence on either α or ϵ. From Crewdson's experimental data, σ_o and γ_o values were calculated using $k = 1.15\, d_p$ (Crewdson et al., 1977), $K = 173$ (Burkett et al., 1971), $\mu = 1.853 \times 10^{-5}$ kg/m · s (Lienhard, 1980), and $\rho_s = 2.6$ g/cm^3 (Brown and Richards, 1970). The several curves entered on the figure were calculated from Eq. 13 and the relationship between σ and σ_o and γ and γ_o of Eqs. 17 and 18, respectively for $\epsilon = 0.4$, 0.45 and 0.5 for several values of $\alpha(1 - \epsilon)/\epsilon$. This method of displaying the results allows us to select the most likely values of ϵ and α to match theory and experiment.

The data in Figure 3 illustrate an unresolved effect of orifice diameter. Four data points appear for each particle size. The four with the largest value of σ_o are for the largest particle diameter, while the four with the smallest value of σ_o are for the smallest particle diameter. The other data group in a similar fashion with decreasing particle size associated with increasing average γ_o for each group. So, for each particle size, the trend with orifice diameter is such as to indicate an increasing flowrate of gas (i.e., $\alpha(1 - \epsilon)/\epsilon$ increases) as the orifice diameter increases. This effect could be a result of a larger orifice pressure drop effect for the smaller orifices, a variation of ϵ with orifice diameter, or simply an inadequate accounting of the physics of the flow by our analysis. Within these limitations, the data allow us to establish ranges for the induced gas flowrate as shown in Table 1.

TABLE 1. IMPLICATIONS OF THEORY ON INTERSTITIAL GAS FLOW FROM CONICAL HOPPERS (CF. FIGURE 3)

$1.0 < \gamma_o$	$\epsilon \simeq 0.5$	$0.8 < \dfrac{\alpha(1 - \epsilon)}{\epsilon} < 0.9$
$0.3 < \gamma_o < 1.0$	$\epsilon \simeq 0.45$	$0.6 < \dfrac{\alpha(1 - \epsilon)}{\epsilon} < 0.8$
$\gamma_o < 0.3$	$\epsilon \simeq 0.4$	$0 < \dfrac{\alpha(1 - \epsilon)}{\epsilon} < 0.7$

As mentioned, Dhoka (1970) reported the results of measurements of the flow of glass beads through sharp-edged orifices in a cylindrical container similar to the familiar egg timer. His apparatus consisted of two 5 cm long by 1.9 cm I.D. cylinders, each closed on one end by an aluminum disc. The two cylinders were joined on their open ends with an orifice-plate spacer. Experiments were run at earth-normal gravity and at elevated gravity on a 64 cm radius centrifuge for orifice sizes of 0.159, 0.238 and 0.318 cm, with particles of nominal sizes of 90, 127, 180, 255 and 360 μm.

The experiments were conducted by first charging one cylinder of the egg-timer assembly with a known mass of particles. For the earth-normal gravity experiments, flow was started by turning over the egg timer so that the gravity vector acted along its axis. In the centrifuge experiments, a release mechanism was contrived to "turn over" the egg timer. The error associated with the starting transient implied by this method of operation was generally negligible. Because the container was gimballed at the end of the centrifuge arm, it swung to an equilibrium angle, and the effective gravitational acceleration was always in the direction of particle flow.

Dhoka measured the flowrate with a stop watch while observing the egg timer as it was illuminated by a strobe light. A provision was made either to seal the egg timer, in which case the pressure difference between the top and bottom container versus time was also measured, or to open both the upper and lower compartments to the atmosphere. In the former case, a countercurrent air flow, equal in volumetric flowrate to that of the particles, acts to impede the particle flow. This situation leads to $\alpha = -1$.

For the sealed container case, Dhoka's measurements showed that the flowrate was independent of time, and consequently independent of the head of material, and that the pressure difference decreased linearly with time. From the open container measurements, Dhoka found the flowrate to vary with g^n, where $n \geq 0.5$. The results are in general agreement with those of Crewdson et al. (1977), discussed above, but a comparable comparison with theory is not warranted since here we have no estimate of either α or ϵ.

Because for the sealed container case we know that $\alpha = -1$, we can compare Dhoka's results with our theory by merely estimating ϵ and using approximate angle of approach values that appear in the literature. The comparison is shown in Figures 4, 5 and 6 where we have plotted the reciprocal particle flowrate, normalized by the reciprocal particle flowrate at earth-normal gravity, against gravity level, measured in units of earth-normal gravity, for three orifice sizes. Discrete data are experimental results taken from Figures 3, 4 and 5 of Dhoka's thesis, and the solid curves were calculated for $\epsilon = 0.5$ and $\beta = 40°$ (Brown and Richards, 1970) from

$$\frac{\dot{m}_{p,e}}{\dot{m}_p} = \left(\frac{g_e}{g}\right)^{1/2} \left(\frac{\sigma_e}{\sigma}\right) \tag{19}$$

where σ_e is the value of σ at earth-normal gravity. Values of K, k and μ are the same as the ones we used above in analyzing Crewdson's data; for ρ_s we chose 2.9 g/cm^3 (Brown and Richards, 1970).

Theoretical and experimental results appear to be in general agreement except for the 180 μm particles. We do not know why the agreement is poor for this one particle size. Possible reasons that come to mind are an incorrect value of ϵ or β, a mechanism change in the discharge phenomenon, and experimental error.

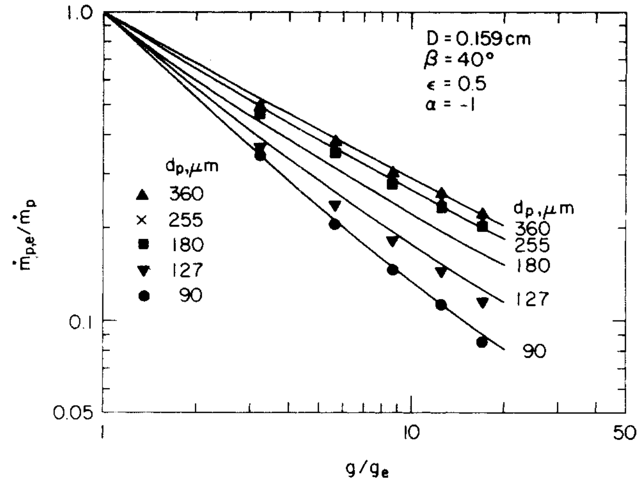

Figure 4. Normalized reciprocal particle flowrate of glass beads in an egg-timer-like device with an orifice of 0.159 cm diameter as a function of gravity level. Data of Dhoka (1970).

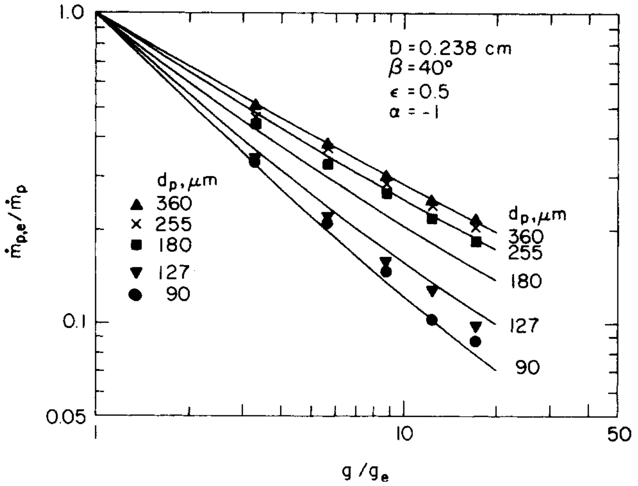

Figure 5. Normalized reciprocal particle flowrate of glass beads in an egg-timer-like device with an orifice of 0.238 cm diameter as a function of gravity level. Data of Dhoka (1970).

Since σ is rather insensitive to β, the exact value of β used is rather unimportant. However, σ is sensitive to the choice for ϵ, particularly for larger values of γ. From Crewdson's results we might expect ϵ to decrease for increasing d_p so that the value used for ϵ for the 180 μm particles should be less than that used for the 90 μm particles. Reducing ϵ below 0.5 for the large particles will cause the theoretical curves to fall below the ones shown, and the discrepancy between theory and experiment will only be magnified.

The last set of experimental data we wish to discuss was obtained by Resnick et al. (1966). They studied the flow of two sizes of sand (d_p = 456 and 680 μm), KCl crystals, and lentils from a plexiglas vessel 14 cm in diameter and 85 cm high through circular orifices of diameters 0.71, 1.00, 1.41 and 2.00 cm. The top of the vessel was sealed, and an air flow in the direction of particle flow was passed through the hopper. Measurements consisted of particle flowrate, air flowrate, and bed pressure drop.

In Figure 7, we show Resnick's sand data as σ versus γ, along with the theoretical curve from Eq. 13, for $\epsilon = 0.5$ and $\beta = 30°$ (Brown and Richards, 1970). Again we used the same values of K, k and μ as we did for Crewdson's data, and we used the particle densities given by Resnick et al. Since Resnick et al. measured both the particle and gas flowrates, α values could be computed from the data. The thirty-nine data points represented on Figure 7 were actually taken from ADI-8876, a compilation of Resnick's raw data. They represent the sand data for which an air flowrate was listed.

Agreement between theory and experiment in Figure 7 is quite good for $\gamma > -1$, where the data cover a range of particle flowrates from about 12 to 265 g/s and a range of air flowrates from about 33 to 863 cm³/s. Even when the experimental results deviate from the theoretical curve for $\gamma < -1$ it seems that a correlation between σ and γ exists. We suspect the discrepancy derives from large particle Reynolds numbers, due to large values of α, at the larger negative γ's. When Re_p is large, the pressure gradient is no longer proportional to the slip velocity, and our theoretical relation between σ and γ must fail.

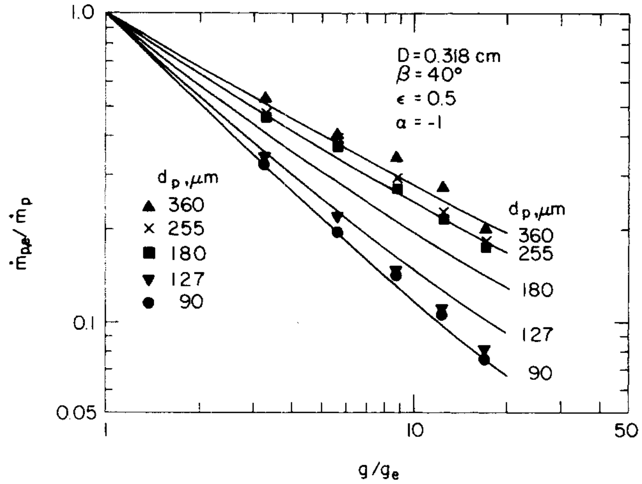

Figure 6. Normalized reciprocal particle flowrate of glass beads in an egg-timer-like device with an orifice of 0.318 cm diameter as a function of gravity level. Data of Dhoka (1970).

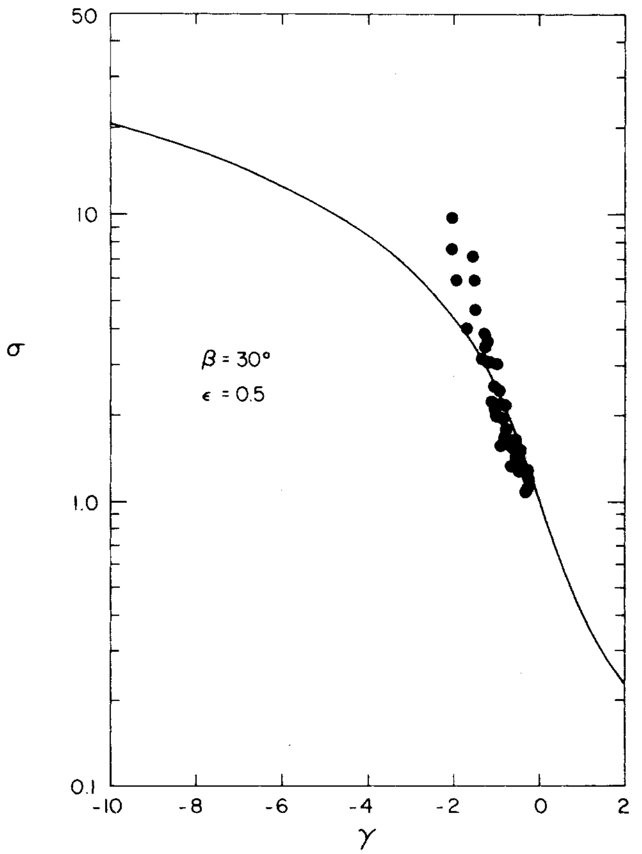

Figure 7. The effect of aiding air flow on the flow of sand from a flat-bottom hopper. Data of Resnick et al. (1966).

An estimate of the particle Reynolds number is:

$$Re_p = \frac{\rho_f \dot{m}_p d_p}{\rho_s \mu 2\pi R_o^2 \epsilon (1 - \cos\beta)} \left| \frac{\epsilon}{1-\epsilon} - \alpha \right| \quad (20)$$

where Re_p is based on particle diameter and an average slip velocity over the flow area at the free-fall arch. For Resnick's data, Re_p is less than about 120 for the 456 μm diameter particles and less than about 470 for the 680 μm diameter particles when γ is greater than -1. Reynolds numbers for the data of Dhoka and Crewdson are estimated to be no larger than about 20 or so.

For the linear relation between pressure gradient and slip velocity to hold, the particle Reynolds number should be less than 20 (Kunii and Levenspiel, 1969). This restriction appears to be met in Dhoka and Crewdson's experiments. For Reynolds numbers greater than 2000, the skin friction responsible for the linear relationship is small compared to the form drag, and the pressure gradient becomes proportional to the square of the slip velocity (Kunii and Levenspiel, 1969). Apparently, for Resnick's results, when γ is less than -1, both skin friction and form drag should be considered in determining the pressure gradient. Hence, our theory underpredicts the flowrate due to an underestimate of the pressure gradient.

The three sets of experimental results discussed above give a convincing but not wholly satisfactory test of the theory. A more comprehensive test would require accurate *in situ* measurement of the void fraction in the vicinity of the orifice, and in the case of the self-induced gas flow studies of Crewdson et al., direct measurements of the gas flowrate as well. Further consideration needs to be given to flows at higher gas flowrates, where a linear relation between pressure drop and slip velocity does not hold.

NOTATION

D = orifice diameter
d_p = particle diameter
e = mechanical energy per unit mass
$F(\epsilon)$ = pressure gradient-slip velocity proportionality factor (Eqs. 11 and 12)
g = acceleration of gravity
g_e = earth-normal acceleration of gravity
K = dimensionless constant (Eq. 12)
k = twice the width of the statistically empty annular region, Figure 1
L = characteristic length
$\dot{m}_f$ = fluid mass flowrate
$\dot{m}_p$ = particle mass flowrate
$\dot{m}_{p,e}$ = particle mass flowrate for $g = g_e$
n = exponent
o = position of the virtual origin, Figure 1
P = pressure
P_f = fluid pressure
P_s = solids pressure
q = heat transfer per unit mass
r = radial distance measured from the virtual origin
R_m = radial position of the free-fall arch, Figure 1
R_o = radial position equal to $(D - k)/(2 \sin\beta)$, Figure 1
Re_p = particle Reynolds number
s = specific entropy
T = absolute temperature
u = specific internal energy
v = speed
v_f = fluid speed
v_s = solids speed
w = work transfer per unit mass

Greek Letters

α = ratio of gas to particle volume flowrates
β = angle of approach
γ = dimensionless parameter reflecting the importance of the fluid phase (Eqs. 15 and 16)
γ_o = $\gamma \dfrac{\epsilon^2}{(1-\epsilon)} \left\{ \dfrac{\alpha(\epsilon - 1) + \epsilon}{\epsilon} \right\}^{-1}$
ϵ = void fraction
$\eta(\theta)$ = proportional to the volume flowrate of gas (Eqs. 9 and 10)
θ = angle from the vertical
$\lambda(\theta)$ = proportional to the volume flowrate of particles (Eqs. 8 and 10)
μ = fluid viscosity
ρ = mixture density
ρ_f = fluid density
ρ_s = particle density
σ = dimensionless flowrate (Eq. 14)
σ_e = dimensionless flowrate when $g = g_e$
σ_o = $(1 - \epsilon)\sigma$
ϕ = half angle of the gas jet

Subscripts

e = earth-normal
f = fluid phase
p = particles
s = solid phase

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